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Relativistic Quantum Field Inertia and Vacuum Field Noise Spectra

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The equivalence statements for quantum scalar field vacuum states that have been used for the thermal-like Hawking effect and Unruh effect are surveyed. An important ingredient in this framework is the concept of a vacuum field noise spectrum, by which one can obtain information about the curvature invariants of classical worldlines (relativistic classical trajectories). It is argued, in the spirit of the free-fall-type universality, that the preferred quantum field vacua with respect to accelerated worldlines should be chosen from the class of all those possessing stationary spectra for their quantum fluctuations. For scalar quantum field vacua there are six stationary cases, as shown by Letaw some time ago, and reviewed here. However, nonstationary vacuum noises can be treated by a few mathematical methods that are mentioned as well. Since the information about the kinematical curvature invariants of the worldlines is of radiometric origin, suggestions are given on the more useful application of such an academic formalism to radiation and beam radiometric standards for high-energy accelerators and in astrophysics. We conclude with a look at related axiomatic quantum field topics and some other recent work.

1. INTRODUCTION

The legendary *gedanken* discovery of *classical free-fall universality* by Galilei [1][2] is an exciting textbook story (the first actual experiments were done in June 1710 at St. Paul's in London by Newton). Starting with the neutron beam experiments of Dabbs *et al.* [3] in 1965, nonrelativistic *quantum free falls* have also been of much interest. 'Free falls' of quantum wavefunctions (wavepackets), i.e., Schrödinger solutions in a homogeneous gravitational field, are mass dependent and therefore closer to Aristotle's fall. Thus, a resumption of the quest for the universality features of free-fall-type phe-

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[2] For an interesting paper on Aristotle's ideas on free fall, where one can see why he was half right irrespective of the medium, see ref. 2.

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nomena in the quantum realm has emerged in recent years. Moreover, interesting insights into the problem of *relativistic quantum field inertia* have been gained as a consequence of the Hawking effect [4, 5] and the Unruh effect [6]. This helped substantially to display the 'imprints' of gravitation in relativistic quantum physics [7]. Natural questions in this context which I take up in this work are, (i) What does 'free fall' really mean in relativistic quantum field theories? (ii) How should one formulate equivalence principles (EPs) for quantum field state? (iii) What are the restrictions on quantum field states imposed by the EPs?

The method of quantum detectors is very useful for the understanding of quantum field inertial features. New ways of thinking of quantum fluctuations have been promoted and new pictures of vacuum states have been provided, of which the landmark one is the heat bath interpretation of the Minkowski vacuum state from the point of view of a uniformly accelerating noninertial quantum detector. Essentially, simple, not to say toy, model particles (just two energy levels separated by E and a monopole form factor), commonly known as Unruh–DeWitt (UDW) quantum detectors of uniform, one-dimensional proper acceleration a in Minkowski vacuum, are immersed in a scalar quantum field 'heat' bath of temperature

$$T_a = \frac{\hbar}{2\pi ck} \cdot a \tag{1}$$

where $\hbar$ is Planck's constant barred c is the speed of light in vacuum, and k is Boltzmann's constant. A formula of this type was first obtained by Hawking [4] in a paper in 1974 black hole explosions, then in 1975 by Davies [8] in a moving-mirror model, and finally by Unruh in 1976 [6]. For first-order corrections to this formula see Reznik [9]. This so-called Unruh temperature is proportional to the lineal uniform acceleration, and the scale of such noninertial quantum field 'heat' effects with respect to the acceleration is fixed, by the numerical values of universal constants, to the very low value of 4×10^{-23} in cgs units. In other words, the huge acceleration of 2.5×10^{22} cm/s^2 can produce a blackbody spectrum of only 1 K. In the (radial) case of Schwarzschild black holes, using the surface gravity $\kappa = c^4/4GM$ instead of a, one immediately gets the formula for their Hawking temperature T_κ. In a more physical picture, the Unruh quantum field heat reservoir is filled with the so-called Rindler photons (Rindler quasiparticles), and therefore the quantum transitions are to be described as absorptions or emissions of the Rindler reservoir 'photons.' Also recall that according to an idea popularized by Smolin [10, 11], one can think of *zero-point fluctuations, gravitation*, and *inertia* as the only three *universal* phenomena of nature. However, one may also think of inertia as related to those peculiar collective quantum degrees of freedom which are the vacuum expectation values (vev's) of Higgs fields.

As we know, these vev's do not follow from the fundamentals of quantum theory. On the other hand, one can find papers claiming that inertia can be assigned to a Lorentz-type force generated by electromagnetic zero-point fields [12]. Moreover, one also has the well-known *Rindler condensate* concept of Gerlach [13]. Amazingly, one can claim that there exist completely coherent zero-point condensates, like the Rindler–Gerlach one, which entirely mimic the Planck spectrum, without any renormalization, as the case is for the Casimir effect.

In this work, I will stick to the standpoint based on the well-known concept of vacuum field noise (VFN) [14]—or vacuum excitation spectrum from the point of view of quantum UDW detectors—because in my opinion this not only provides a clear origin of the relativistic thermal effects, it avoids at the same time uncertain generalizations, and also helps one of my purposes here. This sheds light on the connection between the stationary VFNs and the equivalence principle statements for scalar field theories.

2. SURVEY OF QUANTUM DETECTOR EQUIVALENCE PRINCIPLES

The Unruh picture can be used for interpreting Hawking radiation in Minkowski space [15]. In order to do that, one has to consider the generalization(s) of the EPs to quantum field processes. A number of authors have discussed this important issue with various degree of detail, and with some debate [17–28]. Nikishov and Ritus [29] raised the following objection to the heat bath concept. Since absorption and emission processes occur in finite space-time regions, the application of the local principle of equivalence requires a constant acceleration over those regions. However, the space-time extension of the quantum processes are in general of the order of inverse acceleration. In Minkowski space it is not possible to create homogeneous and uniform gravitational fields having accelerations of the order of a in spacetime domains of the order of the inverse of a.

Grishchuk *et al.* [17] and Ginzburg and Frolov [19] wrote extensive reviews on the formulations of a quantum field equivalence principle (QFEP). One should focus on the response functions of quantum detectors, in particular, the UDW two-level monopole detector in stationary motion. In the asymptotic limit this response function is the integral of the quantum noise power spectrum. Or, since the derivative of the response function is the quantum transition rate, the latter is just the measure of the vacuum power spectrum along the chosen trajectory (worldline) and in the chosen initial (vacuum) state. This is valid only in the asymptotic limit and more realistic cases require calculations in finite time intervals [30]. Denoting by $R_{M,I}$, $R_{R,A}$, $R_{M,A}$ the detection rates with the first subscript corresponding to the vacuum (either Minkowski or Rindler) and the second subscript corresponding to either inertial or accelerating worldline, one can find for the UDW detector in a scalar vacuum that $R_{M,I} = R_{R,A}$, expressing the dissipationless character of the vacuum fluctuations in this case, and a thermal factor for $R_{M,A}$ leading to the Unruh heat bath concept. In the case of a uniform gravitational field, the candidates for the vacuum state are the Hartle–Hawking (HH) and the Boulware (B) vacua. The HH vacuum is defined by choosing incoming modes to be those of positive frequency with respect to the null coordinate on the future horizon and outgoing modes as positive-frequency ones with respect to the null coordinate on the past horizon, whereas the B vacuum has positive-frequency modes with respect to the Killing vector, which makes the exterior region static. For an ideal, uniform gravitational field the HH vacuum can be thought of as the counterpart of the Minkowski vacuum, while the B vacuum is the equivalent of the Rindler vacuum. Then, the QFEP can be formulated in one of the following ways:

Quantum Detector-QFEP: HH–M Equivalence: (i) The detection rate of a free-falling UDW detector in the HH vacuum is the same as that of an inertial UDW detector in the M vacuum.

(ii) A UDW detector at rest in the HH vacuum has the same DR as a uniformly accelerated detector in the M vacuum.

Quantum Detector-QFEP: B–R Equivalence: (iii) A UDW detector at rest in the B vacuum has the same detection rate as a uniformly accelerated detector in the R vacuum.

(iv) A free-falling UDW detector in the B vacuum has the same detection rate as an inertial detector in the R vacuum.

Let us present one more formulation, due to Kolbenstvedt [26]:

Quantum Detector-QFEP: Kolbenstvedt: A detector in a gravitational field and an accelerated detector will behave in the same manner if they feel equal forces and perceive radiation baths of identical temperature.

In principle, since the Planck spectrum is Lorentz invariant (and even conformal invariant) its presence in equivalence statements is easy to accept if one recalls that Einstein EP requires local Lorentz invariance. The linear connection between ‘thermodynamic’ temperature and one-dimensional, uniform, proper acceleration, which is also valid in some important gravitational contexts (Schwarzchild black holes, de Sitter cosmology), is indeed a fundamental relationship because it allows for an absolute meaning of quantum field effects in such *ideal* noninertial frames as soon as one recognize thermodynamic temperature as the only *absolute*, i.e., fully *universal*, energy-type physical concept.

3. THE SIX TYPES OF STATIONARY SCALAR VFNs

In general, the scalar quantum field vacua are not stationary stochastic processes (abbreviated SVES) for all types of classical trajectories on which the UDW detector moves. Nevertheless, the lineal acceleration is *not* the only case with that property, as was shown by Letaw [31, 32][3] who extended Unruh's considerations, obtaining six types of worldlines with SVES for UDW detectors (SVES-1 to SVES-6, see below). These worldlines are solutions of some generalized Frenet equations on which the condition of constant curvature invariants is imposed, i.e., constant curvature κ, torsion τ, and hypertorsion ν, respectively. Notice that one can employ other frames such as the Newman–Penrose spinor formalism, as Unruh [34] did, but the Serret–Frenet one is in overwhelming use throughout physics. The six stationary cases are the following:

1. $\kappa = \tau = \nu = 0$ (inertial, uncurved worldlines). SVES-1 is a trivial cubic spectrum

$$S_1(E) = \frac{E^3}{4\pi^2} \tag{2}$$

i.e., as given by a vacuum of zero-point energy per mode $E/2$ and density of states $E^2/2\pi^2$.

2. $\kappa \neq 0$, $\tau = \nu = 0$ (hyperbolic worldlines). SVES-2 is Planckian, allowing the interpretation of $\kappa/2\pi$ as 'thermodynamic' temperature. In the dimensionless variable $\epsilon_\kappa = E/\kappa$ the vacuum spectrum reads

$$S_2(\epsilon_\kappa) = \frac{\epsilon_\kappa^3}{2\pi^2(e^{2\pi\epsilon_\kappa} - 1)} \tag{3}$$

3. $|\kappa| < |\tau|$, $\nu = 0$, $\rho^2 = \tau^2 - \kappa^2$ (helical worldlines). SVES-3 is an analytic function corresponding to case 4 below only in the limit $\kappa \gg \rho$,

$$S_3(\epsilon_\rho) \xrightarrow{\kappa/\rho\to\infty} S_4(\epsilon_\kappa) \tag{4}$$

Letaw plotted the numerical integral $S_3(\epsilon_\rho)$, where $\epsilon_\rho = E/\rho$ for various values of κ/ρ.

4. $\kappa = \tau$, $\nu = 0$ (the spatially projected worldlines are the semicubic parabolas $y = (\sqrt{2}/3)\kappa x^{3/2}$ containing a cusp where the direction of motion is reversed). SVES-4 is analytic, and since there are two equal curvature invariants, one can use the dimensionless energy variable ϵ_κ,

[3] For the nonrelativistic case see ref. 33.

$$S_4(\epsilon_\kappa) = \frac{\epsilon_\kappa^2}{8\pi^2\sqrt{3}}\, e^{-2\sqrt{3}\epsilon_\kappa} \tag{5}$$

It is worth noting that S_4 is rather close to the Wien-type spectrum $S_W \propto \epsilon^3\, e^{-\mathrm{const.}\epsilon}$.

5. $|\kappa| > |\tau|$, $\nu = 0$, $\sigma^2 = \kappa^2 - \tau^2$ [the spatially projected worldlines are catenaries, i.e., curves of the type $x = \kappa \cosh(y/\tau)$]. In general, SVES-5 cannot be found analytically. It is an intermediate case, which for $\tau/\sigma \to 0$ tends to SVES-2, whereas for $\tau/\sigma \to \infty$ it tends toward SVES-4,

$$S_2(\epsilon_\kappa) \xleftarrow{0 \leftarrow \tau/\sigma} S_5(\epsilon_\sigma) \xrightarrow{\tau/\sigma \to \infty} S_4(\epsilon_\kappa) \tag{6}$$

6. $\nu \neq 0$ (rotating worldlines uniformly accelerated normal to their plane of rotation). SVES-6 forms a two-parameter set of curves. These trajectories are a superposition of the constant linearly accelerated motion and uniform circular motion. The corresponding vacuum spectra have not been calculated by Letaw even numerically.

Thus, only the hyperbolic worldlines having just one nonzero curvature invariant allow for a Planckian SVES and for a strictly one-to-one mapping between the curvature invariant κ and the 'thermodynamic' temperature. On the other hand, in the stationary cases it is possible to determine at least approximately the curvature invariants, that is, the classical worldline on which a quantum particle moves, from measurements of the vacuum noise spectrum.

4. PREFERRED VACUA AND/OR HIGH-ENERGY RADIOMETRIC STANDARDS

There is much interest in considering the magnetobremsstrahlung (i.e., not only synchrotron) radiation patterns at accelerators from the aforementioned perspective [35] at least since the work of Bell and collaborators [36–38]. It is in this sense that a sufficiently general and acceptable statement on the *universal* nature of the kinematical parameters occurring in a few important quantum field model problems can be formulated as follows:

- There exist accelerating classical trajectories (worldlines) on which moving ideal (two-level) quantum systems can detect the scalar vacuum environment as a stationary quantum field vacuum noise with a spectrum directly related to the curvature invariants of the worldline, thus allowing for a radiometric meaning of those invariants.

Although this may look like an extremely ideal (unrealistic) formulation for accelerator radiometry, where the spectral photon flux formula of

Schwinger [39] is very effective, recall that Hacyan and Sarmiento [40] developed a formalism similar to the scalar case to calculate the vacuum stress-energy tensor of the electromagnetic field in an arbitrarily moving frame and applied it to a system in uniform rotation, providing formulas for the energy density, Poynting flux, and stress of zero-point oscillations in such a frame. Moreover, Mane [41] suggested the Poynting flux of Hacyan and Sarmiento to be in fact synchrotron radiation when it is coupled to an electron.

Another important byproduct, and actually one of the proposals I put forth in this paper, is the possibility to choose a class of preferred vacua of the quantum world[4] as *all* those having stationary vacuum noises with respect to the classical (geometric) worldlines of *constant* curvature invariants because in this case one may find some necessary attributes of universality in the more general quantum field radiometric sense [44] in which the Planckian Unruh thermal spectrum is included as a particularly important case. Of course, much work remains to be done for a more "experimental" picture of highly academic calculations in quantum field theory, but a careful look at the literature shows that there are already definite steps in this direction [45–51]. Notice that all the aforementioned scalar quantum field vacua look extremely ideal from the experimental standpoint. Indeed, it is known that only strong external fields can make the quantum electrodynamic vacuum react and show its physical properties, becoming similar to a magnetized and polarized medium, and only by such means can one learn about the physical structure of the QED vacuum. Important results on the relationship between the Schwinger mechanism and the Unruh effect have been reported in recent work [52–61].

5. NONSTATIONARY VFNs

Though the nonstationary VFNs do not enter statements of equivalence type, they are equally important. Since such noises have a time-dependent spectral content, one needs joint time and frequency information, i.e., generalizations of the power spectrum analysis such as tomographic processing [62] and wavelet transform analysis [63; for review see ref. 64]. Alternatively, since in the quantum detector method the vacuum autocorrelation functions are the essential physical quantities, and since according to fluctuation-dissipation theorem(s) (FDT) they are related to the linear (equilibrium) response functions to an initial condition/vacuum, more FDT-type work, especially its generalization to the out-of-equilibrium case [65], will be useful in this framework. One can hope that effective temperature concepts can be introduced following the reasoning already developed for systems with slow

[4] For such a concept in a different context see refs. 42 and 43.

dynamics (glasses) [66]. In fact, there is some progress due to Hu and Matacz [67] in making more definite use of FDT for vacuum fluctuations. Very recently, Gour and Sriramkumar [68] questioned if small particles exhibit Brownian motion in the quantum vacuum and concluded that even though the answer is in principle positive, the effect is extremely small and thus very difficult to detect experimentally.

6. AXIOMATIC QFEPs

At the rigorous, axiomatic level, Hessling [69; see also 70, 71] published further results on the algebraic quantum field equivalence principle (AQFEP) due to Haag and collaborators. Hessling's formulation is too technical to be reproduced here. The difficulties are related to the rigorous formulation of *local position invariance*, a requisite of equivalence, for the singular short-distance behavior of quantum fields, and to the generalization to interacting field theories. Various general statements of locality [72–74] for linear quantum fields are important steps toward proper formulations of AQFEP. These are nice, but technical results coming mainly from clear mathematical exposition involving algebraic-thermal states, namely the Kubo–Martin–Schwinger states of Hadamard type. Hessling's AQFEP formulation is based on the notion of quantum states *constant up to first order* at an arbitrary spacetime point, and means that for these states a certain scaling limit should exist, and moreover a null-derivative condition with respect to a local inertial system around that arbitrary point is to be fulfilled for all *n*-point functions. For example, the vacuum state of the Klein–Gordon field in Minkowski space with a suitable *scaling function* satisfies Hessling's AQFEP. Using as a toy model the asymptotically free ϕ^3 theory in six-dimensional Minkowski space, Hessling showed that the derivative condition of his AQFEP is not satisfied by this interacting quantum field theory, which perturbatively is similar to quantum chromodynamics. This failing is due to the running coupling constant that does not go smoothly to zero in the short-distance limit. If one takes AQFEP or generalizations thereof as a *sine qua non* for physically acceptable quantum field vacuum states, then one has at hand a useful selection guide for even more complex vacua such as the Yang–Mills one [75] or those of quantum gravity [76, 77].

Since the *time–thermodynamics* relation in general covariant theories and the connection with Unruh's temperature and Hawking radiation are an active area of research due to the remarkable correspondence between causality and the modular Tomita–Takesaki theory [78–85], it would be interesting to formulate in this context some sort of AQFEP statement beyond that of Hessling.

Finally, the work of Faraggi and Matone [86] is to be noted, where a sort of mathematical equivalence postulate is introduced stating that all physical systems can be connected by a coordinate transformation to the free system with vanishing energy, uniquely leading to the quantum analog of the Hamilton–Jacobi equation, which is a third-order nonlinear differential equation. The interesting feature of their approach, which they carry on in both nonrelativistic and relativistic domains, is the derivation of a trajectory representation of quantum mechanics depending on the Planck length.

7. CONCLUSIONS

The first conclusion of this work is that considerations of equivalence type in quantum field theories may well guide the abstract research in this area toward the highly required feature of *universality*, which, being an important form of *unification*, is among the ultimate purposes of meaningful theoretical research. This may apply to the act of measuring generic field operators, as was argued by D'Ariano [87] for the homodyne tomography technique in quantum optics.

The second conclusion refers to the hope that Hawking and Unruh effects are not only mathematical idealizations. In particular, their vacuum excitation spectrum interpretation can be used for what one may call high-energy kinematical radiometry, at least as guiding principles in establishing rigorous high-energy and astrophysical radiometric standards. Whether or not the Unruh and Hawking effects really occur [88], they can be employed as a sort of standard in relativistic quantum field radiometry.

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